Government Engineering College, Bakhtiyarpur, Buxar & Bhojpur

Assignment (Mathematics)

Branch:- Civil of 2nd semester.

Answer the following Questions

1. (a) By using Calculus of Residue evaluate the following.

(i) $\int_0^\infty \frac{\cos 2x}{x^2+1}\,dx$ (ii) $\int_0^\infty \frac{\sin mx}{x}\,dx$ (iii) $\int_0^\infty \sin x^2\,dx$

(iv) $\int_0^\infty \frac{dx}{x^6+1}$ (v) $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$

(b) Evaluate the following

(i) $\int_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)^2(z-2)}\,dz$, where C is the circle $|z|=3$

(ii) $\int_C z^2 e^{1/z}\,dz$, where C is the circle $|z|=1$

(iii) $\int_C \frac{\sin^2 z}{(z-\pi/6)^2}\,dz$, where C is the circle $|z|=2$.

2. (a) Obtain Cauchy Riemann eqn in Cartesian form. If $f(z)$ is a holomorphic function of z, prove that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2+\left\{\frac{\partial}{\partial y}|f(z)|\right\}^2=|f'(z)|^2$$

(b) Find the Analytic function $f(z)=u+iv$, if

$$u+v=\frac{2\sin 2x}{e^{2y}-e^{-2y}-2\cos 2x}$$

(c) Determine the bilinear transformation that maps the points $1-2i,\ 2+i,\ 2+3i$ respectively into $2+2i,\ 1+3i,\ 4$.

3 (a) Using method of Variation of Parameters solve $(D^2+a^2)y=\sec ax$

(b) Solve $(px-y)(x+py)=2p$

(c) Prove that $\int_{-1}^{1} x^2 P_{n-1}(x)P_{n+1}(x)\,dx=\frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$

(d) Prove that $J_{-3/2}(x)=-\sqrt{\frac{2}{\pi x}}\left(\sin x+\frac{\cos x}{x}\right)$

4 (a) Solve $p\cos(x+y)+q\sin(x+y)=z$

(b) Solve $\frac{\partial^2 z}{\partial x^2}-3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=e^{2x-y}+e^{x+y}+\cos(x+2y)$.

5(a) Obtain Simpson's $3/8^{th}$ rule for Numerical integration and Evaluate $\int_0^6 \frac{dx}{1+x^2}$, by using

(i) Trapezoidal rule (ii) Simpson's $1/3^{rd}$ rule (iii) Simpsons $3/8^{th}$ rule.

(iv) Wedle's rule.

(b) Use Milne's Predictor-Corrector method to obtain the solution of the equation $dy/dx = x - y^2$ at $x = 0.8$, given that $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$.
